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Stress Intensity Factors at the tip of a Surface Initiated Crack caused by different Contact Pressure Distributions

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Abstract

Current practice in pavement design is to use a uniformly distributed load over a circular contact patch. However, this is not the reality; tyre-pavement contact stress distributions are very complex. There are vertical, longitudinal and transverse stress components that affect the response of the pavement. The formation of surface initiated cracks is known to be caused by a number of factors, but primarily by the traffic loading, these cracks propagate from the surface into the pavement. The study investigated the Stress Intensity Factors (SIF) of mode one and mode two cracking at the tip of a short crack and as such their influence on the continued propagation further into the pavement. The CAPA-3D finite element software was used to model the two scenarios of uniformly distributed loading and uniformly distributed loading with symmetrical inward shear. This analysis was performed for two different subbase moduli. The analysis was 2D and used plane strain conditions. The loading was moved to a number of different positions relative to the crack for both load cases. The study showed that the SIF for both K_I (stress intensity factor for mode one cracking) and K_{II} (stress intensity factor for mode two cracking) was dependant on the load type and also on its location relative to the crack tip. It also showed the increase in the ratio of asphalt modulus to subbase modulus increases the SIF values at all locations. It was shown that the shearing force only had a noticeable impact on the SIF values when the distance from the crack tip was small or zero.

Keywords: Surface cracking; Contact pressures; Finite element

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1. Introduction

Almost all of the current and past flexible pavement design methods determine the fatigue life by considering the tensile strain or stress at the bottom of the asphalt layers (Agency, 2006; Theyse et al., 1996). These methods assume that fatigue cracks are initiated at the bottom of the bound layers and propagate upwards until the surface of the pavement. It is assumed that the maximum tensile strain occurs at the base of the bound layers directly under the load and the maximum compressive strain is at the top of the pavement under the loading (Perret, 2003). However, the impact of top-down cracking is becoming more widely recognised and has been the subject of applied asphalt research (Baladi et al, 2003; Uhlmeyer et al, 2000; Myers and Roque, 1998). The process of longitudinal surface initiated cracking is usually parallel to the direction of traffic and is in the vicinity of the wheel paths.

Collop and Cebon (1995) used Linear Elastic Fracture Mechanics (LEFM) to model surface cracking due to combined thermal and traffic loading. They used a Finite Element (FE) program to calculate the stress distribution in the un-cracked pavement structure and superimposed the effect of the crack using a Green's function approach. They found that the horizontal contact stress produced high horizontal tensile stresses in a small region on the pavement surface at the edge of the tyre which decayed rapidly with depth. Consequently, a surface-initiated crack was found to grow, although it was not predicted to grow far into the pavement structure (10mm or so). However, there was no allowance for ageing of the surface or the lateral position of the load with respect to the crack which are likely to be significant factors.

Myers and Roque (2001) expanded this approach by using a FE analysis that uses more realistic three-dimensional contact stress distributions for radial truck tyres. They found that surface-initiated crack propagation was primarily a tensile failure and that analysing crack growth with realistic load spectra was critical. The most significant factor was found to be the position of the load with respect to the position of the crack. In Figure 1 below shows how the K_I factor varies for the position of the loading relative to the crack and the length of the crack. When the load is applied away from the crack, the intensity factor is negative were as with the crack directly under the tyre the stress intensity factor is positive and then becomes negative for crack lengths greater than 20mm.

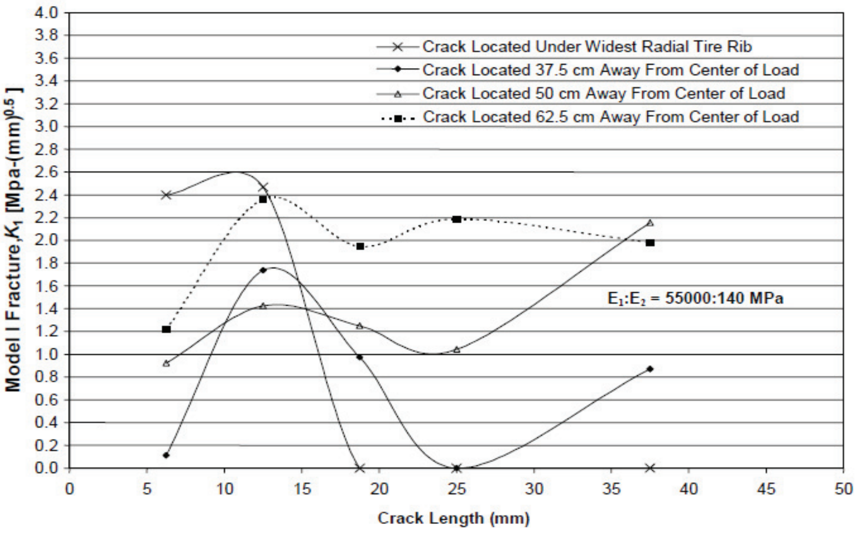

Figure 1 Variation of K_1 with crack depth and load position (Meyers and Roque, 2001)

Analytical solutions for cracked structures only exist for a limited number of idealised structures such as infinite plates with central cracks. Practical engineering problems on the other hand, like a multilayer pavement structure with surface cracks, have complex geometries and require the flexibility offered by numerical solutions such as finite elements to carry out this kind of analysis. This paper sets out to show the variation in the Stress Intensity Factor at a predefined 10mm crack on the surface of a pavement using a 2D finite element mesh assuming plane strain conditions. The loading was moved relative to the crack and the variation in the SIF was calculated and recorded. The nature of the loading was changed from a uniform vertical load to a uniform vertical load with symmetrical inward shear. The analysis with the same loading was also carried out using two different subbase moduli.

2. Procedure

2.1 Mesh Set-up

The meshing of a crack-tip is a very particular operation in finite element theory as the crack tip represents a stress singularity of $(1/\sqrt{r})$ order (Anderson, 1995). This means that the stresses and strains in the vicinity of a crack tip are inversely proportional to the square root of the radial distance from the crack tip regardless of the configuration of the cracked structure. The earliest attempts of applying finite elements to evaluate the stresses and strains at a crack tip used conventional elements. This however, was not successful as these elements cannot accurately represent the singularity even with highly refined meshes and higher order polynomial elements (Tong and Pian, 1973). The next solution which was developed was special elements to represent the singularity. The idea behind the method was to combine the analytical solution, which is accurate near the crack tip, with the finite element solution which is accurate elsewhere. There were numerous types with different shapes, some were cracked and others uncracked with the crack tip located at one of their corners (Benzley, 1974; Walsh, 1971; Byskov, 1970).

To overcome the problems of the conventional and special elements, another singular element called a quarter point element was derived (Barsoum, 1976; Henshall and Shaw, 1975). The QP elements are derived by distorting the shape function in order to create a singular field at the crack tip within the domain of the element. The inverse square root singularity can be simulated in a quadratic isoparametric element by a simple shifting of the mid side nodes near the crack tip to the quarter points and the merging of nodes to create a triangular element of 6 nodes. An example of this can be seen in figure 2 below.

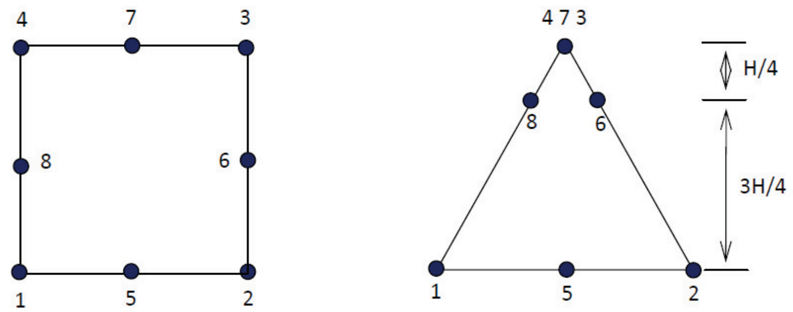

Figure 2 Standard 8-Noded Element and Collapsed Quarter Point Element

This is the method that is employed in this study for the investigation of the SIF near the crack tip. The crack being modelled is a 10mm deep crack with a 1.4mm opening at the surface reducing to a point at 10mm from the surface. There are 8 QP elements of the same size arranged around the crack tip as can be seen in Figure 3.

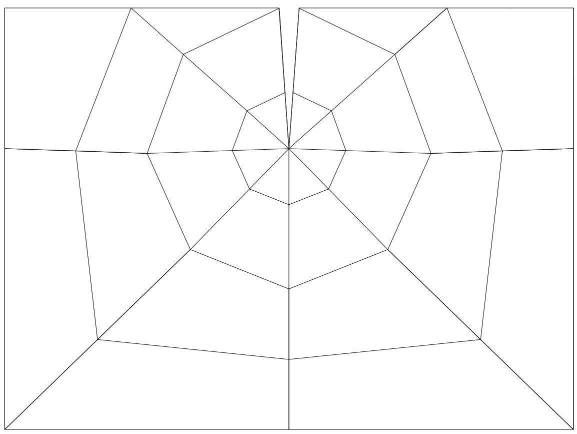

Figure 3 Finite Elements around the crack tip

Then transitional elements are used outside this where their mid side node is shifted to βL from the crack tip. In Q-1 for βL the length of the QPE is defined as 1 and L is the length from the crack tip to the outside of the transitional element. The transitional elements mid side nodes were adjusted as to reflect the square root singularity on the stress and strains at the tip of the crack. The mesh outside this is then made up of 8 noded quadratic 2-D isoparametric elements that are merged with a global mesh of the pavement structure. The global mesh is restrained in the horizontal and vertical direction at the base and the sides are restrained in the horizontal direction.

$$\beta L = \frac{L+2\sqrt{L}+1}{4} \qquad \text{(Q-1)}$$

The thickness of the layers is based on conventional design of pavements. The asphalt layer is 150mm thick with a Young's modulus of 5000MPa and a Poisson ratio of 3.0. The base layer is 300mm in thickness with a modulus for the first analysis of 300MPa and a value of 150MPa for the second analysis, the Poisson value remains constant at 0.3. The subgrade layer then has a Young's modulus of 100MPa and a Poisson ratio of 0.3.

2.2 Loading Arrangement

The first loading scenario is a uniformly distributed load of 666kPa distributed over 300mm in 2D space. This load then is moved from a distance of 450mm from its centre point in steps of 40mm until the edge of the loaded area aligns with the edge of the crack tip. The required measurements for the calculation of the SIF were taken for each step and recorded to show how it varies with distance from the centre of the loading to the crack tip. The second loading scenario was to use the same uniformly distributed loading in conjunction with a symmetrical lateral force of 130kPa applied at the ends of the contact area and the same series of runs were repeated for this load arrangement. This would show the effect of the lateral contact pressure on the SIF and thus its effect on the potential propagation of surface cracking. This was repeated for both modulus ratios.

2.3 Stress Intensity Factor Calculation

The SIF for both the Mode I (opening crack mode) and Mode II (shearing crack mode) were calculated using equations Q-2 and Q-3 respectively. These equations are derived for plane strain conditions and for use with the quarter point element method of calculation. A spreadsheet was set up to analysis the output of the displacements of the nodes around the crack tip and produces the SIF for each loading step and scenario. Then to compare the different loading scenarios and the different loading steps, the results from the spreadsheet were graphed to illustrate their effect.

$$K_I = \frac{G\sqrt{2\pi}}{\sqrt{r}(4-4\mu)}[4(u_d - u_b) + (u_c - u_e)] \qquad \text{(Q-2)}$$

$$K_{II} = \frac{G\sqrt{2\pi}}{\sqrt{r}(4-4\mu)}[4(v_d - v_b) + (v_c - v_e)] \qquad \text{(Q-3)}$$

Where r is the distance from the crack tip to outer most nodes of the quarter point elements on the crack faces. G is the shearing elastic modulus = ($E/2(1+\mu)$) for isotropic elements. μ is Poisson's ratio and u, v are the x and y displacement at the nodes on the crack face. The subscripts of the equation are, (a) is the crack tip node, (b) and (d) are the mid-sided nodes and (c) and (e) are the nodes at the end of the element on the crack face.

3. Results and Discussion

The analysis was run as described earlier and the results were plotted on two separate graphs to highlight the variation of SIF with the location and type of loading. This was done for both modulus ratios; the High Ratio refers to the combination of an asphalt modulus of 5000MPa and subbase modulus of 150MPa. The Low Ratio refers to the combination of an asphalt modulus of 5000MPa and subbase modulus of 300MPa. The distances shown are for the centre of the loaded area to the tip of the crack in the horizontal direction e.g. a distance of 150mm is the edge of the loaded area aligned to the edge of the crack. The two graphs deal with the K_I and K_{II} separately as they are of different magnitudes and do not lend themselves to plotting on the same graph.

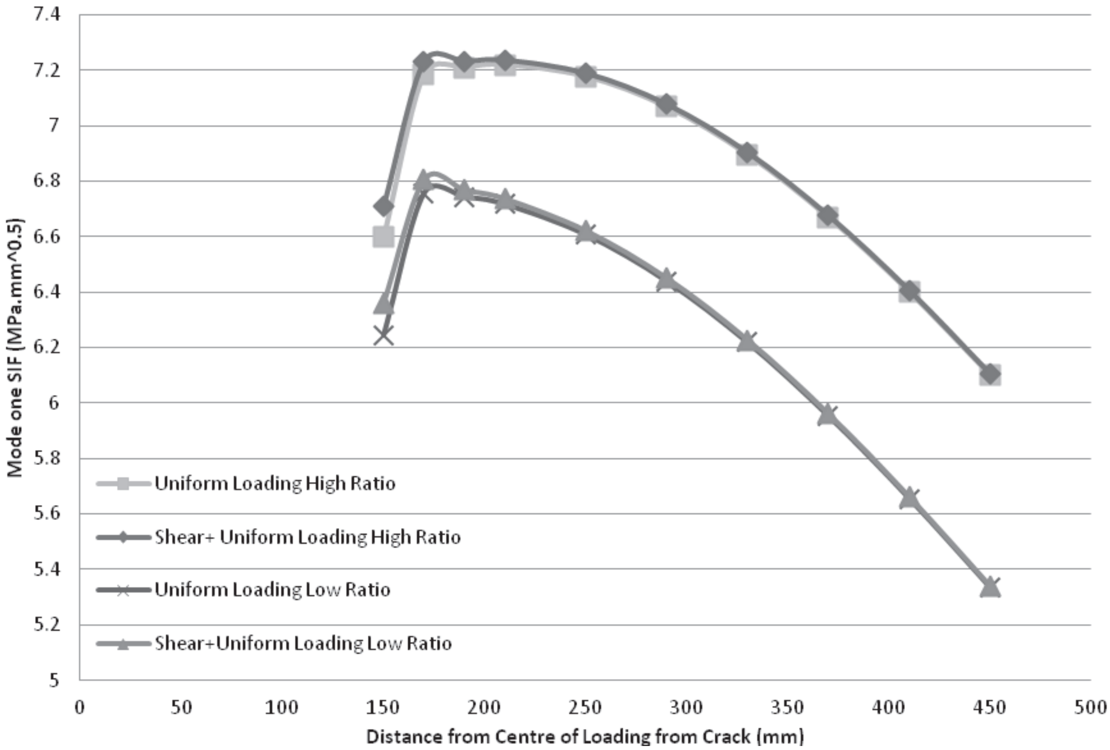

Figure 4 K_I v distance of centre of loading to crack tip

In Figure 4 the K_I SIF is dealt with in relation to the load position, type of loading and the modulus ratio. It can be seen that the further the centre of the loading is from the crack, the lower the SIF that is calculated indicating it has less of an effect on the crack. As the loading approaches the crack in steps of 40mm the SIF increases in a relatively linear way up until a distance of 300mm. Then the slope starts to change and becomes shallower in nature up until it starts to behave differently closer to the edge of the crack. The 150mm distance from the centre of loading to the edge of the crack, aligns the edge of the loaded area and the edge of the crack. At this distance the loading changes from its maximum of 6.76 for the uniform loading low ratio, 6.8 for the shear and uniform loading low ratio to 6.25 and 6.36 respectively. The difference between the two loading types is at its greatest at this reading. This was expected as the effect of the shearing is heavily influenced by the distance from the crack. There are minor changes in the preceding readings that show that the shearing is influencing the SIF calculation as it approaches the edge of the crack.

The trends for the High Ratio as for the Low Ratio are largely the same; however the magnitude of the values at each point is higher by around 0.2. This shows the influence that the ratio of the subbase/asphalt modulus has on the value of the Mode one SIF. This is also particularly striking as the modulus of the material being cracked does not change; it is the subbase modulus that varies. It shows that the behaviour of the crack tip is influenced by the overall behaviour of the pavement layers and not just the asphalt layer. Even with the change in ratio, the effect of the type of loading has on the SIF is broadly the same for both ratios just with the aforementioned general difference of 0.2 between High and Low ratio. Overall, the change in ratio effects the magnitude of SIF but not the effect that loading has on the SIF.

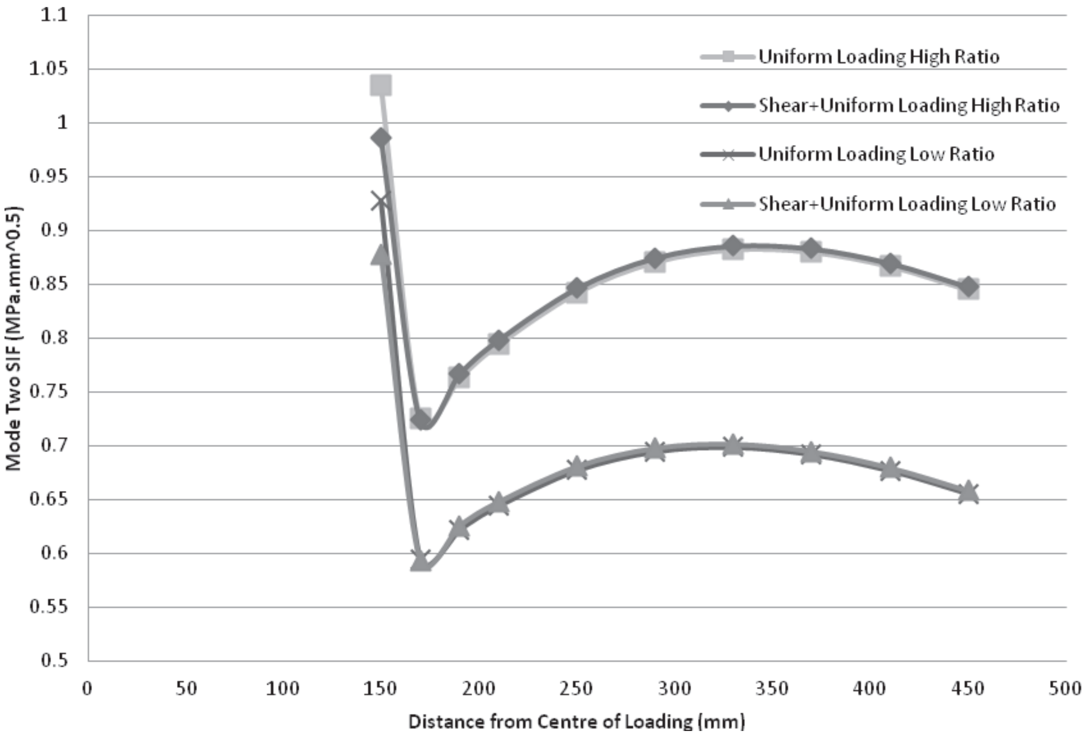

Figure 5 K_{II} v distance to centre of loading to crack tip

In Figure 5 the K_{II} SIF is dealt with in relation to the load position, type of loading and the modulus ratio. This factor is much less than the K_I SIF values reported in Figure 4 for both the high and low modulus ratios and has a different trend with distance from the crack tip. The loading starts at the furthest distance 0.66 and then increases to 0.7 and then reduces back down until it hits its minimum value at 0.59 at a distance 170mm for the low ratio values. The SIF factors for both the loading scenarios shoots up to 0.93 and 0.88 for the uniform and shearing low ratio loading, respectively. Interestingly the K_{II} SIF for the uniform loading at this distance is greater than the shearing and uniform loading. The higher modulus ratio values are higher across all the values. The loading starts at the furthest distance 0.85 and then increases to 0.89 and then reduces back down until it hits its minimum value at 0.72 at a distance 170mm for the low ratio values. The SIF factors for both the loading scenarios shoots up to 1.03 and 0.99 for the uniform and shearing low ratio loading, respectively. The decrease from the 0.89 value to the minimum value is sharper than the low ratio values for both loading scenarios.

There is a relatively constant difference between the high and low ratio of around 0.18. This shows the influence that the ratio of the subbase/asphalt modulus has on the value of the Mode two SIF. Like with the Mode one SIF values this is a significant difference as the modulus of the material being cracked does not change; it is the subbase modulus that varies. It shows that the behaviour of the crack tip is influenced by the overall behaviour of the pavement layers and not just the asphalt layer for both modes of cracking. Even with the change in ratio, the effect of the type of loading has on the SIF is broadly the same for both ratios like the mode one cracking SIF. Overall, the change in ratio effects the magnitude of SIF but not the effect loading has on the SIF. Both graphs show that the loading type has an influence on the respective SIF values if the distance from the loading to the crack is small.

4. Conclusions:

The analysis undertaken in this paper has led to the following conclusions:

- Both SIF's are influenced by the distance of the load from the crack tip but in different ways as can be seen in Figures 4 and 5. The K_I value increasing until the last reading and then decreases. The K_{II} value increases to a point at 300mm and then reduces until it increases rapidly for the last reading.
- The effect of the addition of shearing to the loading can be seen in both analyses. However, this effect only has a visible effect on the SIF values when the distance to the crack tip is small. The largest variation taking place when the edge of the loading is at the edge of the crack.
- The K_I values are circa 10 times the K_{II} values for both the high and low modulus ratios as shown in the graphs. This shows the mode of cracking with the greatest effect on potential crack propagation is the K_I cracking mode as it is higher.
- In the K_I graph, the value decreases steeply when the edge of the loading aligns with the edge of the crack, conversely the K_{II} increases sharply at this position showing the influence the position has on the SIF value for the different cracking modes.
- The variation in the subbase modulus and by consequence the ratio in subbase modulus to asphalt modulus has a relatively large influence on the SIF value for both Mode one and Mode two cracking SIF.
- Even though the magnitude of values in general increases for the high ratio from the low ratio values it has little influence on the behaviour with regard to the loading type for both cracking modes.

This paper gives an insight into the effect of the position of loading and the type of loading has on the K_I and K_{II} SIF values for a predefined 10mm long crack in the surface of a pavement structure. It also shows what effect changing the subbase modulus and by consequence the modulus ratio has on the SIF for both mode one and two cracking SIF's. The analysis gives a good illustration of the variation of SIF with distance, load type, modulus ratio and cracking mode for a 10mm predefined crack for this pavement arrangement.

5. Acknowledgements:

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